Relative demand for different skill groups determines the relative wage structure. The standard framework of production given different sets of labor skills assumes

1. Output y is given by $y = f(K, h(L_1, L_2, ...))$, where $f(K, L) = AL^{\alpha}K^{1-\alpha}$
2. Return on capital r is exogenous
3. $h(L_1, L_2, ...)$ has a nested CES structure

Under the first two assumptions, we have

$$\frac{\partial y}{\partial K} = (1-\alpha)AL^{\alpha}K^{-\alpha} = r$$

$$\Rightarrow K = L\left(\frac{(1-\alpha)A}{\alpha}\right)^{1/\alpha} \text{ and } \frac{y}{K} = \frac{r}{(1-\alpha)} \tag{1}$$

Equation 1 shows that K adjusts to match the overall supply of total labor units L, keeping y/K constant and keeping K/L on a trend path that is driven by the rate of growth of TFP. These assumptions are very plausible at a local level (or for "small open economies") that that the price of capital as exogenous).

Substituting for K, we get

$$y = AL^{\alpha}K^{1-\alpha} = A^{1/\alpha}\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} L \tag{2}$$

which is linear in L. Equation 2 shows that under assumptions 1-3, we can ignore capital.

To analyze the effects of relative supply or relative technology changes (i.e., the part of technology embedded in $h()$), we need to specify the labor aggregator function. A good starting point is a 2-group CES model:

$$L = h(L_1, L_2) = (\theta_1 L_1^{\frac{\sigma-1}{\sigma}} + \theta_2 L_2^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{3}$$

where θ_1 and θ_2 are possibly trending over time. The marginal product of group 1 is

$$h_1(L_1, L_2) = \theta_1 L_1^{\frac{-1}{\sigma}}(\theta_1 L_1^{\frac{\sigma-1}{\sigma}} + \theta_2 L_2^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}} = \theta_1 L_1^{\frac{-1}{\sigma}} L^{\frac{1}{\sigma}} \tag{4}$$

Likewise,

$$h_2(L_1, L_2) = \theta_2 L_2^{\frac{-1}{\sigma}} L^{\frac{1}{\sigma}} \tag{5}$$

Assuming $w_1/w_2 = h_1/h_2$ (i.e., MRTS=relative wage), we have:

$$\log\frac{w_1}{w_2} = \log\frac{\theta_1}{\theta_2} - \frac{1}{\sigma}\log\frac{L_1}{L_2} \tag{6}$$

The slope of the relative demand curve is $-\frac{1}{\sigma}$, which is 0 if the two types are perfect substitutes, and something larger otherwise.

This simple model is widely used to discuss "skill-biased technical change" (SBTC). In the traditional SBTC literature (e.g., Katz and Murphy, 1992), it is assumed that

$$\log\frac{\theta_{1t}}{\theta_{2t}} = a + bt + e_t \tag{7}$$

[1] Thanks to David Card for providing access to his lecture notes. This is a slightly modified excerpt of Card's Lecture 7

leading to a model for the relationship of relative wages to relative supplies:

$$\log \frac{w_{1t}}{w_{2t}} = a + bt - \frac{1}{\sigma} \log \frac{L_{1t}}{L_{1t}} + e_t \tag{8}$$

Freeman (1976) and Katz and Murphy (1992) estimate models of this form, using 2 "types" of labor - high-school equivalents and college equivalents. Dropouts are assumed to be perfect substitutes for HS graduates with a relative efficiency of (roughly) 70%. Post-graduates are assumed to be perfect substitutes for college graduates with a relative efficiency of (roughly) 125%. People with 1-3 years of college are assumed to represent 1/2 unit of HS labor and 1/2 unit of college labor. (There are different conventions about whether supply should be based on the total numbers of adults in each education group, or total employees. There are also different ways to combine men and women). The "magic number" is $\frac{1}{\sigma} = 0.7$, which implies $\sigma = 1.4$ (See KM, equation 19, page 69). It has turned out to be hard to get a model like (8) to work as well as it did in KM's study (and in Freeman, 1976) when the sample is extended to the 1990s and 2000's. Katz and Goldin (2008) present some estimates that have trend breaks in the last two decades and manage to get estimates in the range of $\frac{1}{\sigma} = 0.7$.