Graduate Public Finance
Capital Taxes

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Lecture 5a

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Outline

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Motivation

Equity-efficiency tradeoffs of capital taxation seem especially stark

1. **Efficiency**
   - Capital taxes reduce scale of economic activity
   - Capital accumulation, which may be highly responsive to rates of return, is correlated with economic growth
   - Capital mobility: taxes can lead to misallocation

2. **Equity**
   - Distribution of capital income is much more unequal than labor income
   - Capital mobility: burden may be shifted to labor

3. **Evidence**
   - Empirical evidence/our understanding of capital taxation is less well developed than labor income taxation

4. **Policy Relevance**
   - Future of fiscal policy (taxing robots, driverless cars, corp tax reform)
   - Destination-based cash flow taxes, international reforms

What is capital income?

Individuals derive market income (before tax) from **labor** and **capital**: $z = wl + rk$ where w is wage, l is labor supply, k is wealth, r is rate of return on wealth

1. **Labor income inequality** is due to differences in working abilities (education, talent, physical ability, etc.), work effort (hours of work, effort on the job, etc.), and luck (labor effort might succeed or not)

2. **Capital income inequality** is due to differences in wealth k (due to past saving behavior and inheritances received), and in rates of return r (varies dramatically over time and across assets)

Level and distribution of capital income (1/2)

- Labor income $wl \simeq 75\%$ of national income z
- Capital income risk $rk \simeq 25\%$ of national income z (has increased in recent decades)
- Wealth stock $k \simeq 400\% - 500\%$ of national income z (is increasing)
- Rate of return on capital $r \simeq 5\%$
- $\alpha = \beta \cdot r$ where $\alpha = rk/z$ share of capital income and $\beta = k/z$ wealth to income ratio
- In GDP, gross capital share is higher (35%) because it includes depreciation of capital ($\simeq 10\%$ of GDP)
- National Income = GDP – depreciation of capital + net foreign income

The top 1% share in the US: wealth vs. labor income

Source: Piketty, Saez and Zucman (2016).

Types of wealth and capital income

Definition: Capital Income = Returns from Wealth Holdings

- Aggregate US **Private** Wealth $\simeq$ 4*Annual National Income
- **Housing:** residential real estate (land+buildings) [income = rents] net of mortgage debt
- **Unincorporated business assets:** value of sole proprietorships and partnerships [income = individual business profits]
- **Corporate equities:** Value of corporate stock [income = dividends + retained earnings]
- **Fixed claim assets:** Currency, deposits, bonds [income = interest income] minus debts [credit card, student loans]
- **Pension funds:** Substantial amount of equities and fixed claim assets held indirectly through pension funds

Aggregate Household Wealth

Total household wealth (to national income)
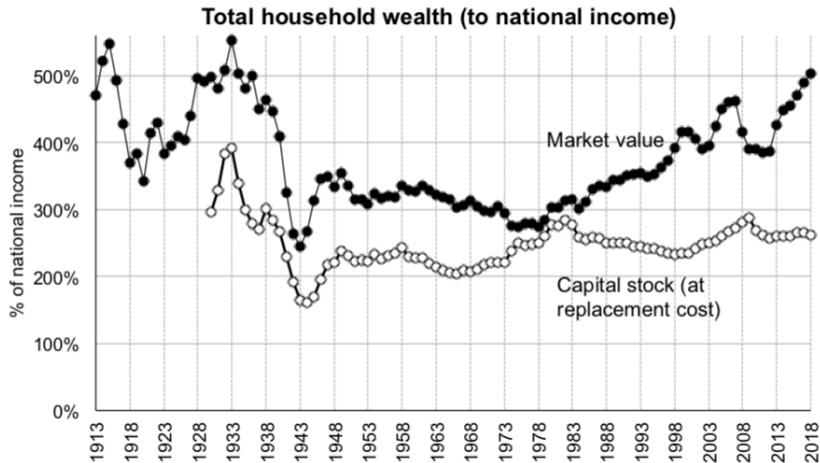
This figure depicts the share of total household wealth relative to national income Source: Piketty, Saez, and Zucman (2018).

Source: Saez Zucman (2019)

Components of Aggregate Household Wealth

A. Components of Aggregate Household Wealth

Legend:
- C-corporation Wealth
- Fixed Income Assets
- Pass-through Business
- Housing (Net of Mortgages)
- Pensions
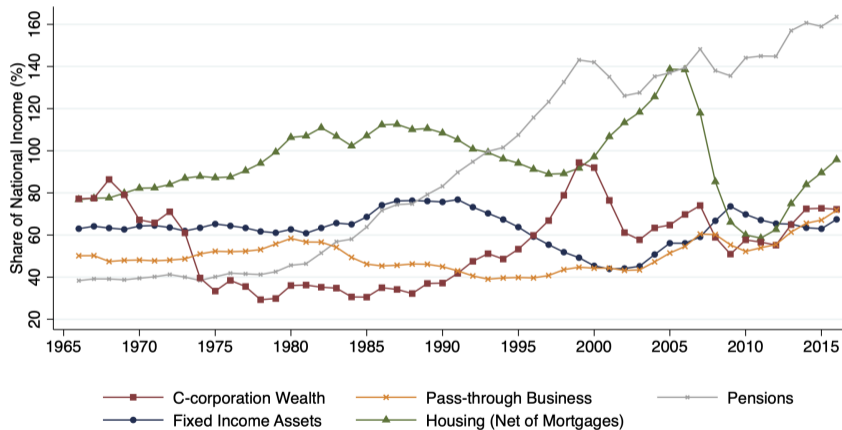
Source: Smith Zidar Zwick (2020)

Components of Aggregate Fiscal Capital Income

B. Components of Aggregate Fiscal Capital Income
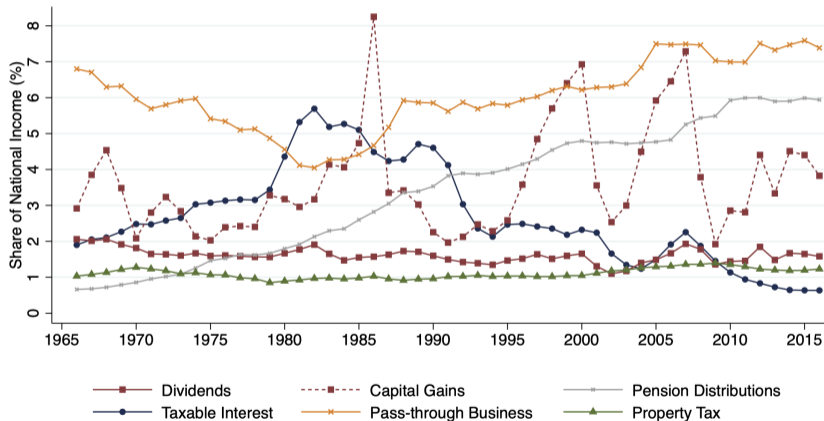
Legend:
- Dividends
- Taxable Interest
- Capital Gains
- Pass-through Business
- Pension Distributions
- Property Tax

Source: Smith Zidar Zwick (2020)

From taxable to total capital income

Retained earnings

Corporate income tax

Nonfilers & other

Income paid to pensions & insurance

Imputed rents + property tax

Dividends, interest, rents & profits reported on tax returns

% of national income

Source: Appendix Table I-S.A8.

FIGURE I

From Taxable Income to National Income (1916–2014)

Sources of wealth and capital income

Sources of wealth and capital income

Wealth $= W$, Return $= r$, Capital Income $= rW$

$$W_t = W_{t-1} + r_t W_{t-1} + E_t + I_t - C_t$$

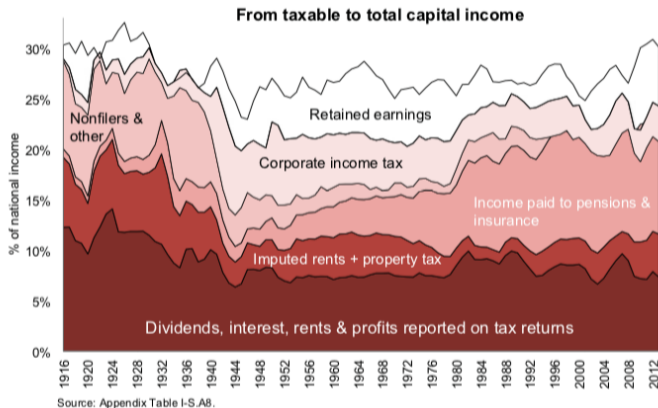where W_t is wealth at age t, C_t is consumption, E_t labor income earnings (net of taxes), r_t is the average (net) rate of return on investments and I_t net inheritances (gifts received and bequests - gifts given).

Differences in Wealth and Capital income due to:

1. Age
2. past earnings, and past saving behavior $E_t - C_t$ [life cycle wealth]
3. Net Inheritances received I_t [transfer wealth]
4. Rates of return r_t

Life Cycle Model
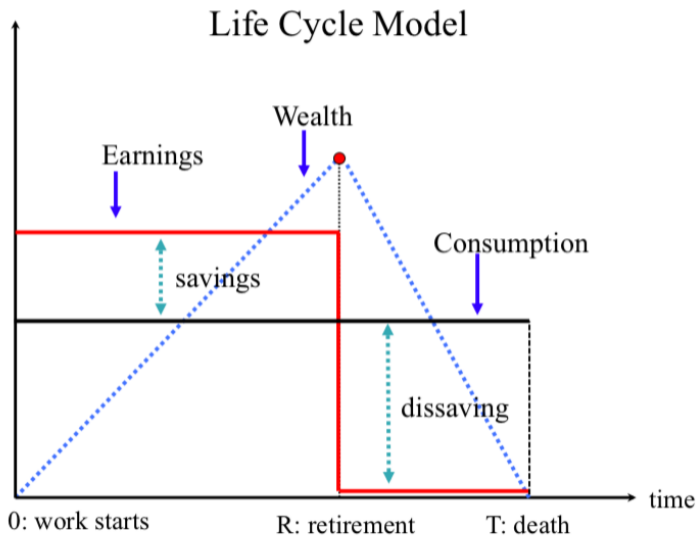
Life cycle wealth versus Inherited wealth

1. **Life-cycle wealth** is wealth from savings earlier in your life
   - (e.g., pension contributions out of earnings, paying down a home mortgage, etc.)
2. **Inherited wealth** is wealth from inheritances received
   - (e.g., receiving a house or a trust fund from parents)

- Distinction matters for taxation because individuals are responsible for life-cycle wealth but not inherited wealth [meritocracy vs. aristocracy]
- Inherited wealth used to be very large in Europe (before World-War I), became small in post-World War II period, but is growing in recent decades (especially in Europe) Piketty (2014)

Analyzes income, wealth, inheritance data over the long-run:

- Growth rate g = population growth + growth per capita. Population growth will converge to zero, growth per capita for frontier economies is modest (1-1.5%) $\Rightarrow$ long-run $g \simeq 1 - 1.5\%$

- Long-run aggregate wealth to income ratio (β) = savings rate (s) / annual growth (g): Proof: $W_{t+1} = (1 + g) \cdot W_t = W_t + s \cdot Y_t \Rightarrow W_t/Y_t = s/g$
  With $s = 8\%$ and $g = 2\%$, $\beta = 400\%$ but with $s = 8\%$ and $g = 1\%$, $\beta = 800\% \Rightarrow$ Wealth will become important

Piketty (2014) book: Capital in the 21st Century

- Rate of return on wealth $r \simeq 5\%$ significantly larger than g [except exceptional period of 1940s-1960s]
- With $r >> g$, role of inheritance in wealth grows and wealth inequality increases [past swallows the future]
    - Explanation: Rentier who saves all her return on wealth accumulates wealth at rate r bigger than g and hence her wealth grows relative to the size of the economy. The bigger $r - g$, the easier it is for wealth to "snowball": fortunes are created faster and last longer
- $\Rightarrow$ Capital income taxation reduces r to $r \cdot (1 - \tau_K) \Rightarrow$ reduces wealth concentration and relative weight of inherited wealth

How much is it taxed?

In the US, total capital taxes can be decomposed into three categories of roughly equal importance from a macro perspective:

1. Corporate tax = 3% of Y (around 20% of a 15% tax base)
2. Annual property rates = 3% of Y (around 1% of a 300% tax base)
3. Personal taxes on a capital income = 2.8% of Y (around 30% of a 15% × 60% = 9% tax base) + estates = 0.2% of Y (around 2% of a 10% tax base)

Won't be able cover all of these in as much depth as I'd like, especially on optimal capital taxation and classic results like Chamley Judd [or Straub Werning (2019)'s reassessment].

I strongly encourage you to review Stancheva's capital tax lecture slides:
`http://scholar.harvard.edu/files/stantcheva/files/capital_taxation.pdf`.[1]

Taxation of capital income by type

1. **Corporate Income Tax** (fed+state): 21% Federal tax rate on profits of corporations [complex rules with many industry specific provisions]: effective tax rate lower.
2. **Individual Income Tax** (fed+state): taxes many forms of capital income
   - Realized capital gains and dividends receive preferential treatment (to lower double taxation of corporate profits)
   - Imputed rent of home owners and returns on pension funds are exempt
3. **Estate tax:** tax on very large estates (40% tax above $11m) bequeathed to heirs (now very small and poorly enforced)
4. **Property taxes** (local) on real estate (old tax):
   - Tax varies across jurisdictions. About 0.5% of market value on average
   - Won't be able to discuss land taxation or housing subsidies in detail, but big deal/important area [see Henry George's Progress and Poverty, which sold millions of copies (second only to Bible in 1890s) and helped spark Progressive Era].

Proposals to reform current taxation of wealth and capital income

1. **Wealth**
   - Estate tax on inheritances
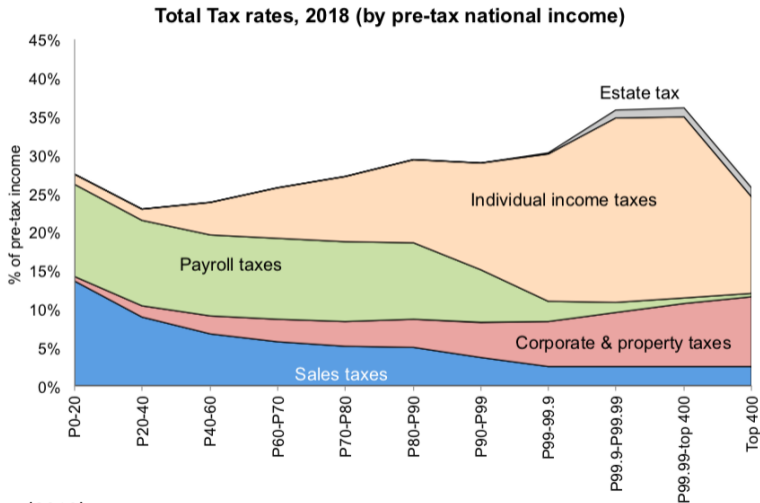   - Local property tax
2. **Capital income**
   - Corporate tax
   - Individual income tax

But some cite concerns:

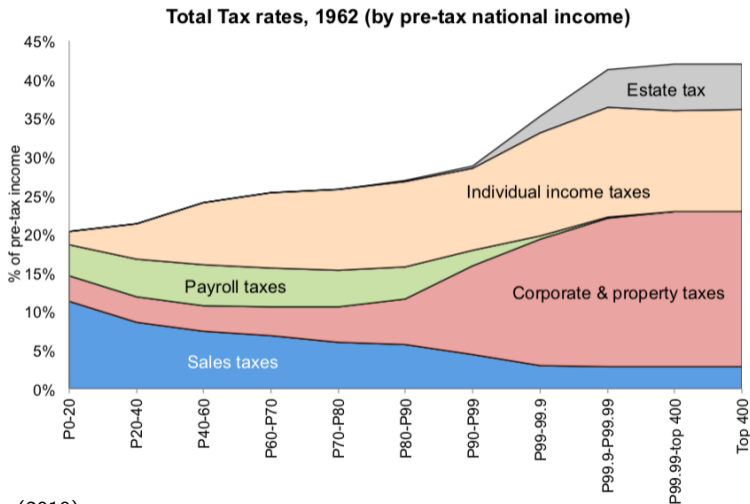- Estate tax avoidance concerns, property tax not very progressive
- Low corporate tax rate (21%) and lack of integration $\Rightarrow$ Rich will incorporate and accumulate within corporations
- Realized capital gains tax partly retained earnings and pure K gains but with loopholes (deferral and step-up of basis after transfer/inheritance)

Total Tax rates, 2018 (by pre-tax national income)

Source: Saez Zucman (2019)

Total Tax rates, 1962 (by pre-tax national income)

Estate tax

Individual income taxes

Payroll taxes

Corporate & property taxes

Sales taxes

% of pre-tax income

Source: Saez Zucman (2019)

Democrats' Emerging Tax Idea: Look Beyond Income, Target Wealth

Lawmakers and 2020 candidates offer a range of options focused on capturing some of the trillions of dollars in assets belonging to the nation's richest

Source: https://www.wsj.com/articles/democrats-emerging-tax-idea-look-beyond-income-target-wealth-11560916571

A related proposal of accrual taxation

- **Mark-to-market:** tax gains as they accrue. Assets valued every year, and taxpayers pay taxes on the gain or deduct the loss
- **Retroactive accrual:** tax gains upon sale. Minimize benefit of deferring sale by including deferral charge equivalent to back taxes due with interest
- **Combination approach:** mark-to-market for publicly traded assets and retroactive accrual for non-publicly traded assets (harder to price annually)

Proposed accrual tax plans

- **Sen. Ron Wyden**
  - Combination approach: mark-to-market and retroactive accrual
  - Applied only to top earners ($\geq$ \$1 million in annual income) and top wealth-holders ($\geq$ \$10 million in assets for three consecutive years, with some exemptions)
  - Use ordinary-income tax rates, no specified top rate
  - Use revenues to fund Social Security

- **Joe Biden**
  - Tax unrealized gains at death, abolishing stepped-up basis
  - Double income-tax rate on capital gains (currently 20%) for taxpayers with income $\geq$ \$1 million
  - Revenues delayed relative to other plans

Rethinking Capital Gains Taxation

Democrats are looking at major changes to the way capital gains taxation works. The effects of their tax proposals would depend on each taxpayer's circumstances and on market performance.

CURRENT LAW
Assets can appreciate without capital gains taxes and heirs pay taxes only on gains in value after the original owner's death.

BIDEN PLAN
Death would be considered a realization event, triggering capital gains taxes on appreciated assets, paid at ordinary income tax rates.

WYDEN PLAN
Each year, investors would pay income taxes on the gain in their assets. This is called a mark-to-market system.
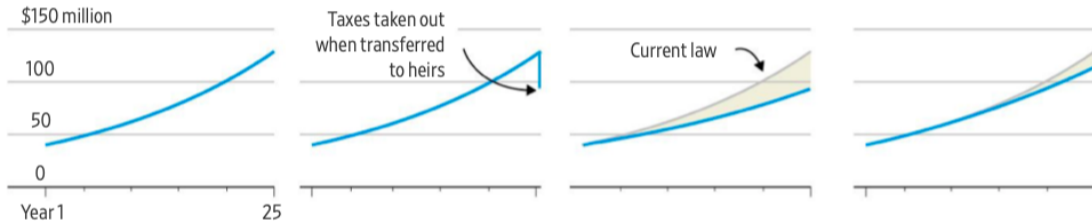
WARREN PLAN
Net worth above $50 million subject to a 2% annual tax, plus a 1% tax on net worth above $1 billion.

Source: https://www.wsj.com/articles/democrats-emerging-tax-idea-look-beyond-income-target-wealth-11566916571

Example one: Asset value begins at $40 million, 5% growth until person dies in year 25

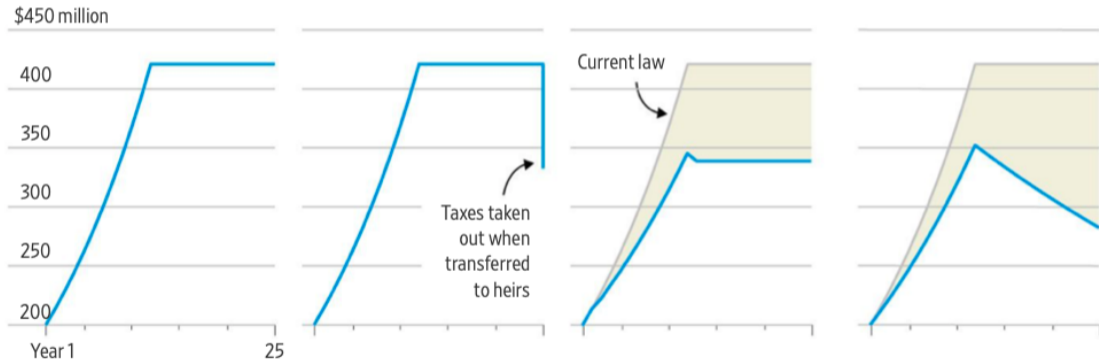Total taxes taken under law/plans:

$0   **$35.6 million**   **$21.4 million**   **$11.9 million**

Source: https://www.wsj.com/articles/democrats-emerging-tax-idea-look-beyond-income-target-wealth-11566916571

Example two: Asset value begins at $200 million, 7% growth until year 12, person dies in year 25

Total taxes taken under law/plans:

$0   **$88.4 million**   **$58.3 million**   **$121.2 million**

Source: https://www.wsj.com/articles/democrats-emerging-tax-idea-look-beyond-income-target-wealth-11566916571

Outline

Impact of a Capital Tax

Impact of a Capital Tax
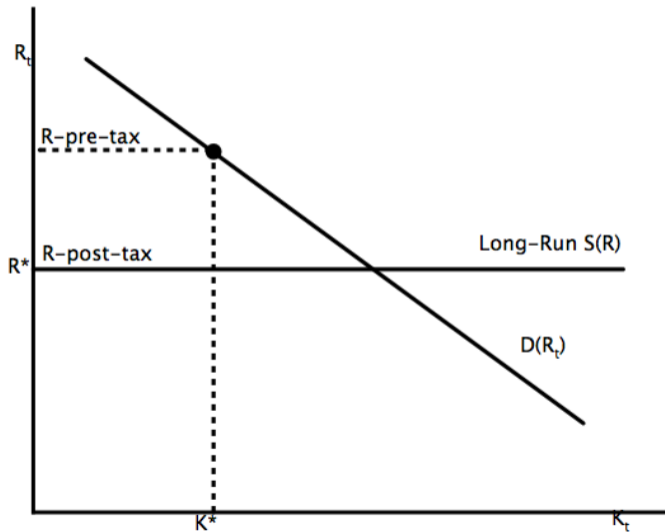
Who bears the capital tax in the long run?

- Who gets the triangle above R-pre-tax (i.e., consumer surplus in the typical S and D graph)?
- If firms don't earn profits, this all goes to workers in terms of higher wages or lower prices
- A key object is the **elasticity of capital supply**, is likely larger (and some think infinite) in the LR
- Note that the distortion in the capital market reduces surplus more than it increases tax revenues (as with most taxes)
- Finally, distortions due to capital taxation are often considered in a dynamic context in which the distortion compounds overtime (See Werning Straub (2019) on the classic Chamley-Judd results)

Preliminaries: use and asset markets for capital

Capital markets: 4 key equations

We will use 4 equations to analyze capital markets

1. **Stock Adjustment:** the amount of capital today depends on how much there was yesterday, depreciation, and new investment

2. **Asset pricing equilibrium**:[2] the rental price of using an asset is simply the cost of buying the good and re-selling it after one period

3. **Rental market equilibrium:** the demand for using capital services is downward sloping

4. **Investment market equilibrium:** the supply of capital assets is upward sloping

See https://home.uchicago.edu/cbm4/cpt/videos.html chapter 15 for more discussion

Rental and asset markets are linked

Use the link between rental and asset markets to analyze capital markets

Rental Market   Asset Market
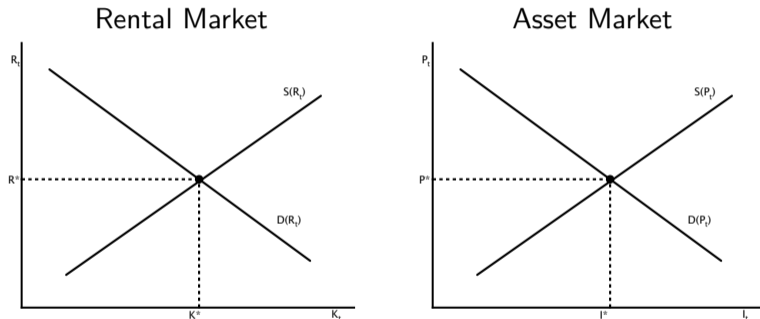
where R_t is the **rental price** of using capital services K_t and P_t is the **purchase price**, which depends on the level of investment I_t.

4 key equations

1. **Stock Adjustment:** $K_t = (1 - \delta)K_{t-1} + I_t$

2. **Asset pricing equilibrium** The rental cost of using an asset is simply the cost of buying the good and re-selling it after one period

3. **Rental market equilibrium:** $K = D(R)$

4. **Investment market equilibrium:** $I = S(P)$

2. Asset pricing equilibrium (without taxes)

What is the relationship between rental and capital prices?

The rental cost of using an asset is simply the cost of buying the good and re-selling it after one period

$$R_t = P_t - \frac{(1-\delta)P_{t+1}}{1+r}$$

- r is the nominal rate of interest
- P_{t+1} is next year's price for the good

2. Asset pricing equilibrium: Housing example

Suppose

- Suppose $r = .10$ and $\delta = 0$
- $P_{t+1} = \$\ 110$ K
- $P_t = \$\ 100$ K
- What is R_t?

2. Asset pricing equilibrium: Housing example

Suppose

- Suppose $r = .10$ and $\delta = 0$
- $P_{t+1} = \$ 110$ K
- $P_t = \$ 100$ K
- What is R_t?

$$R_t = P_t - \frac{(1-\delta)P_{t+1}}{1+r}$$
$$R_t = 100 - \frac{110}{1+.1}$$
$$R_t = 0$$

2. Analyzing Rental Price

We can rearrange the expression to show rental prices depend on three things:

$$R_t = \frac{rP_t + \delta P_{t+1} + P_t - P_{t+1}}{1 + r}$$

1. Interest cost[3]: rP_t
2. Depreciation: δP_{t+1}
3. Market re-evaluation: $P_t - P_{t+1}$

Rental prices are higher, the higher is r, the greater is the physical rate of depreciation, and the faster the price of the asset is declining

2. Analyzing Rental Price: Car example

$$R_t = \frac{rP_t + \delta P_{t+1} + P_t - P_{t+1}}{1 + r}$$

- If cars lose their value quickly (i.e., $P_t >> P_{t+1}$), then rental prices will be pretty high

2. Analyzing Capital Prices

We can also use the rental price expression to calculate the implied capital price

$$P_t = R_t + \frac{R_{t+1}(1-\delta)}{(1+r)} + \frac{R_{t+2}(1-\delta)^2}{(1+r)^2} + ...$$

- This equation can be obtained by recursively substituting for future prices in the rental price equation
- This equation should look familiar to you (prices are PV of cash flow stream)
- Capital prices are higher when rental payments to the owner are large and soon

3. Rental Market Equilibrium for Housing Services

$$K_t = D(R_t)$$

- The demand for housing services depends on the flow cost of housing services (i.e., the rental rate R_t). R_t is what I pay to use the asset
- Housing services are provided by the stock of housing K_t
- The demand side of the market links the current rental price and the current stock

3. Rental Market Equilibrium
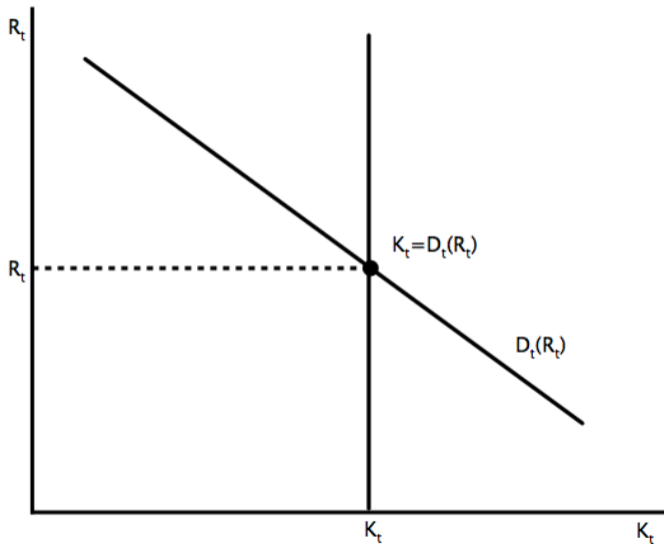
4. Investment Market Equilibrium

$$I_t = S(P_t)$$

- The supply of new construction, investment depends on its current price
- Think of this as a new car producer who decides how much to supply based on the current price
- Alternatively, housing construction firms see high house prices and build. They build more when prices are high.

4. Investment Market Equilibrium
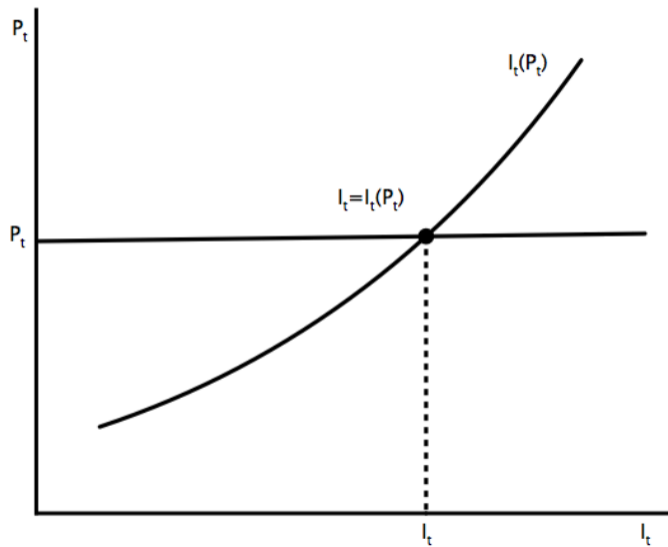
$$K_t = (1 - \delta)K_{t-1} + I_t \tag{1}$$

$$R_t = P_t - \frac{(1 - \delta)P_{t+1}}{1 + r} \tag{2}$$

$$K_t = D(R_t) \tag{3}$$

$$I_t = I(P_t) \tag{4}$$

4 equations and 4 unknowns, but depends on past and the future. Where do past and future come in?

- When we look at a market equilibrium for the housing market at any one point in time, we must realize that today's market is influenced by both the past and future
- The effect of the past comes through the effect of past production decisions on the stock of housing
- The effect of the future comes from the effect of future expected rental rates on the current price

$$\bar{K} = (1 - \delta)\bar{K} + \bar{I}$$
$$\bar{R} = \bar{P} - \frac{(1 - \delta)\bar{P}}{1 + r}$$
$$\bar{K} = D(\bar{R})$$
$$\bar{I} = S(\bar{P})$$

What does the system look like in steady state?

$$\bar{I} = \delta \bar{K}$$

$$\bar{R} = \bar{P} \left(1 - \frac{(1 - \delta)}{1 + r} \right)$$

$$\bar{K} = D(\bar{R})$$

$$\bar{I} = S(\bar{P})$$

We can use the first two equations to plug into the second two equations and obtain the supply and demand in the use market.

$$\bar{I} = \delta \bar{K}$$

$$\frac{\bar{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)} = \bar{P}$$

$$\bar{K} = D(\bar{R})$$

$$\underbrace{\bar{I}}_{\delta \bar{K}} = S(\underbrace{\bar{P}}_{\frac{\bar{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)}})$$

$$\bar{K} = D(\bar{R})$$

$$\bar{K} = \frac{1}{\delta} S \left(\frac{\bar{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)} \right)$$

This shows that we have a familiar supply and demand diagram where the quantity is K and the price is R

Capital Market Equilibrium
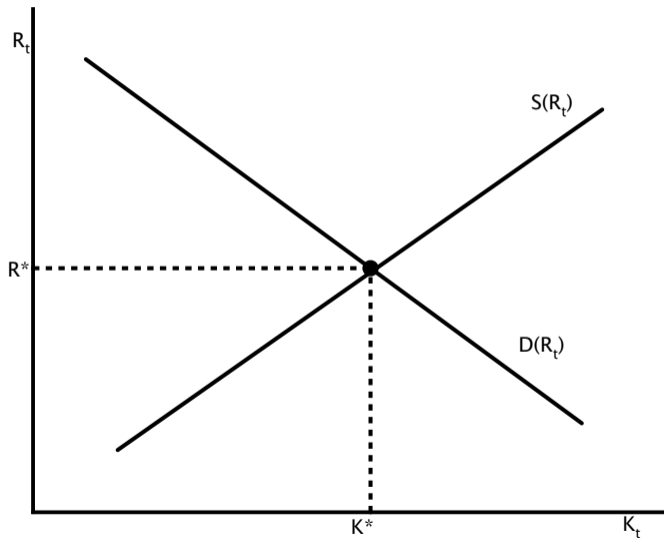
Earthquake Destroys part of capital stock
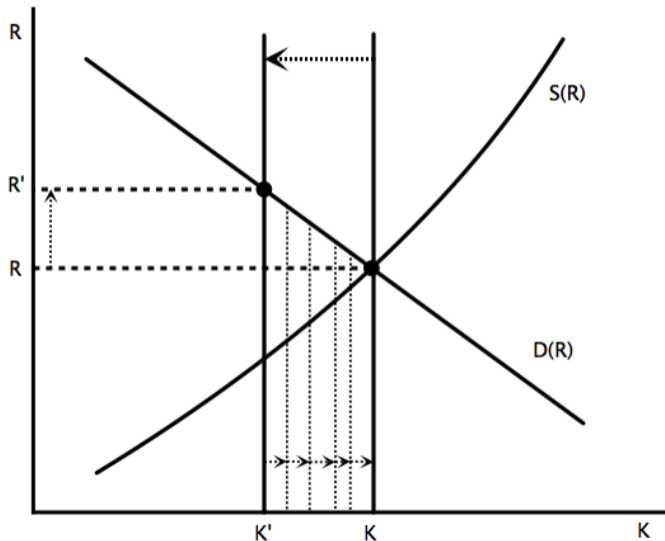
Earthquake Destroys part of capital stock

- The main impact is on the use market. Lower K increases R.
- Higher rental prices cause the asset price P to increase.
- However, since rental rates we decline as we rebuild capital stock, the increase in P is smaller than increase in R
- Investment follows P, so it will jump and slowly decline as we rebuild the stock
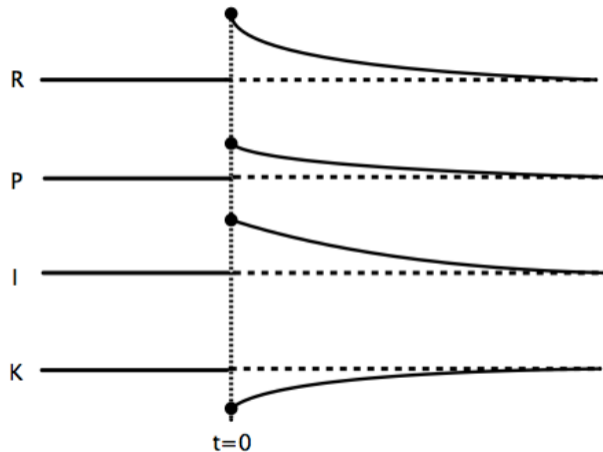
Speed of Adjustment

What determines the speed of convergence to the steady state?

1. **Elasticity of demand** in the rental market ε^D. For example, the more the rental price goes up following a destruction of the capital stock, the faster we will converge to steady state (since it will make the capital price go up more, and thereby also investments). With a higher elasticity (in absolute value), the rental price will go up more.

2. **Elasticity of supply** in the investment market ε^S. This will make investment go up more when the capital price goes up.

3. The **depreciation rate** δ. This may be the most important aspect, since it puts a lower bound on the speed of convergence. The slowest rate at which the economy ever can return to the steady state is δ.

Others examples: ◄ construction costs   ◄ interest rates   ◄ housing bubble?

$$\dot{K}(t) = -\delta K(t) + I(t) \tag{5}$$

$$P(t) = \int_t^\infty e^{-(r+\delta)(z-t)} R(z) dz \tag{6}$$

$$K(t) = D(R(t)) \tag{7}$$

$$I(t) = I(P(t)) \tag{8}$$

Sometimes we use $R(t) = (r + \delta)P(t) - \dot{P}(t)$, which is implied by the second expression. Note, that this expression, however, does not imply the 2nd expression b/c it doesn't include a boundary condition that says prices must be set at the correct level.

Combining equations

$$\dot{K}(t) = -\delta K(t) + I(P(t))$$
$$K(t) = D((r + \delta)P(t) - \dot{P}(t))$$
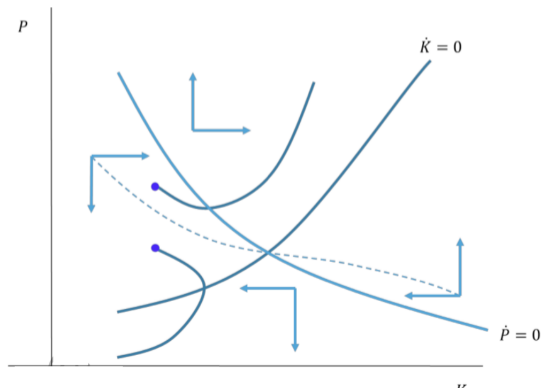
Phase diagram

- Above the $\dot{K}(t) = 0$ line, the price is high, so the capital stock is increasing (so arrow above $\dot{K}(t) = 0$ points to the right
- To the right of the $\dot{P}(t) = 0$ line, the level of capital stock is too high, so the price is too low and is therefore rising overtime (so we have an up arrow).
- Convergence to the steady state happens along the saddle path

See https://home.uchicago.edu/cbm4/cpt/videos.html chapter 16 for more discussion

Taxes, Housing, and Asset Markets (Poterba, 1984)

Poterba, QJE 1984
"Tax subsidies to owner-occupied housing: an asset market approach"

- Q: what is the effect of changes in inflation and tax policy on housing market (prices and quantities)
- The paper develops and estimates an asset-market model of the US housing market
  - Starts with the demand for housing services as a function of tax policy and inflation
  - Then solves for asset prices
  - Then adds supply side to incorporate how quantities change when asset prices change
  - Calibrates demand side and estimates supply side and then runs simulations/counterfactuals
- Finds as much as 30 percent of the increase in the real price of owner-occupied structures could be attributed to the user cost decline of the late 1970s

I'll present a related model/framework from Poterba's AEA slides (notation is a bit different than 1984 paper)

Equilibrium in the use market for housing services

For a single period t, equilibrium in the housing services market is defined as:

$$\underbrace{R(H_t)}_{\text{Value of asset services per unit of housing}} = \underbrace{q_{H,t}(r(1-\tau)+\delta) - (q_{H,t+1} - q_{H,t})}_{\text{Investor cost of holding house for one period}} \qquad (9)$$

$$q_{H,t} = \text{house price at start of period t}$$
$$q_{H,t+1} - q_{H,t} = \text{capital gain or loss during period t}$$
$$r(1-\tau) = \text{opportunity cost of funds (i.e., after-tax interest rate)}$$
$$\tau = \text{marginal income tax rate}$$
$$\delta = \text{depreciation}$$

Source: Jim Poterba, 2013: Public Economics AEA slides

Add in a tax

If a tax of required payment T is levied on houses in period t, equilibrium becomes:

$$R(H_t) - T_t = q_{H,t}(r(1-\tau) + \delta) - (q_{H,t+1} - q_{H,t}) \tag{10}$$
$$= q_{H,t}(1 + r(1-\tau) + \delta) - q_{H,t+1} \tag{11}$$

We can rewrite this as

$$q_{H,t} = \frac{R(H_t) - T_t + q_{H,t+1}}{1 + r(1-\tau) + \delta} \tag{12}$$

Solve for house prices (1/2)

We assume perfect foresight so that we can solve by substituting recursively for $q_{H,t+1}$:

$$q_{H,t} = \left(\sum_{t=0}^{S} \frac{R(H_{t+i}) - T_{t+i}}{\left(1 + r(1-\tau) + \delta\right)^{i+1}} \right) + \frac{q_{H,t+S}}{\left(1 + r(1-\tau) + \delta\right)^{S+1}} \qquad (13)$$

We also impose a tranversality condition so that the house price won't "explode" (and so we can drop this term):

$$\lim_{S \to \infty} \frac{q_{H,t+S}}{\left(1 + r(1-\tau) + \delta\right)^{S+1}} = 0$$

Solve for house prices (2/2)

For $S = \infty$, we can rewrite our expression for $q_{H,t}$:

$$q_{H,t} = \sum_{t=0}^{\infty} \frac{R(H_{t+i})}{\left(1 + r(1-\tau) + \delta\right)^{i+1}} - \underbrace{\sum_{t=0}^{\infty} \frac{T_{t+i}}{\left(1 + r(1-\tau) + \delta\right)^{i+1}}}_{\substack{\text{Present discounted value of} \\ \text{current and future tax payments}}} \qquad (14)$$

Effect on house prices depends on supply side

What does this model tell us about changes in house prices over time? Two possibilities:

- **Housing stock is fixed:** We can determine $\Delta q_{H,t}/\Delta T_{t+1}$ from equation 14.
- **Housing stock is endogenous:** higher taxes $\rightarrow$ lower house prices $\rightarrow$ decline in housing construction $\rightarrow$ decreased future stock of housing capital $\rightarrow$ rise in house prices

To formalize the latter case, we must model the supply function for new construction.

Supply side: new home construction increases in house prices

For I_t denoting the gross construction of new housing, we declare

$$I_t = \Psi(q_{H,t}) \tag{15}$$

So, the net change in the housing stock over a period is given by:

$$H_{t+1} - H_t = \Psi(q_{H,t}) - \delta H_t \tag{16}$$

We can rearrange 10 to see that the corresponding change in house prices over a period is:

$$q_{H,t+1} - q_{H_t} = q_{H,t}\big(r(1-\tau) + \delta\big) - R(H_t) + T_t \tag{17}$$

Taxes and housing market dynamics

Equations 16 and 17 show that the steady state is defined by two equations:

$$\Psi(q_H) = \delta H \tag{18}$$

$$q_H\big(r(1-\tau) + \delta\big) = R(H) - T \tag{19}$$

We can graph this steady state along with the impact of a tax increase:
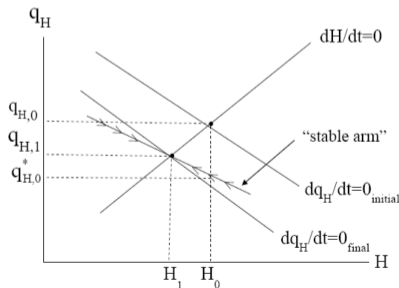
Applying a more realistic user cost framework

To illustrate how this framework can be used to evaluate changes to housing tax policy (pre-TCJA), consider the user cost of owner-occupied housing:

$$c = \left(1 - \tau_y\right)\left(\gamma r_M + (1 - \gamma)r_{Alt} + \beta\right) + m + (1 - \tau_y)\tau_{prop} - \pi_e \qquad (20)$$

τ_y = marginal inc. tax rate for
    mortgage int. and prop.
    tax deducts, invest. inc.

τ_{prop} = property tax rate

γ = home's loan-to-value ratio

r_M = mortgage interest rate

r_{Alt} = return on non-house assets

m = depreciation + maintenance costs

β = Pre-tax housing risk premium

π_e = expected rate of nominal
    house price appreciation

Calibrating user costs and effects of tax changes

For 2012, some plausible parameters are

$$\tau_y = 0.25 \qquad\qquad r_M = r_{Alt} = 0.04$$
$$\tau_{prop} = 0.015 \qquad\qquad m = \beta = 0.02$$
$$\gamma = 0.75 \qquad\qquad \pi_e = 0.02$$

This implies a user cost c of 0.056. Eliminating the federal income tax deduction for the property tax would increase c to 0.06.

How would this affect house prices?

Model	Implied Δ housing prices
Constant housing stock, elasticity of demand of -1	-6.7%
Endogenous stock, calibrated rational expectations model	-3.7%

Simple spatial model: One factor, two locations

Impact of Capital Tax: One factor, two locations

Setup

1. One factor (capital)
2. Two locations: east and west
3. Capital market in each location
4. Total K fixed in economy overall
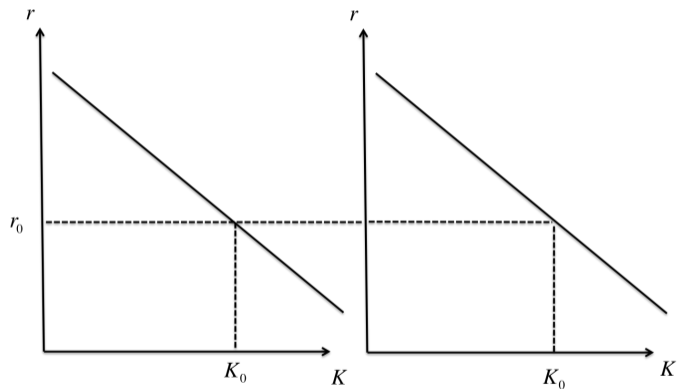
Tax in west

Causes capital to flee to east
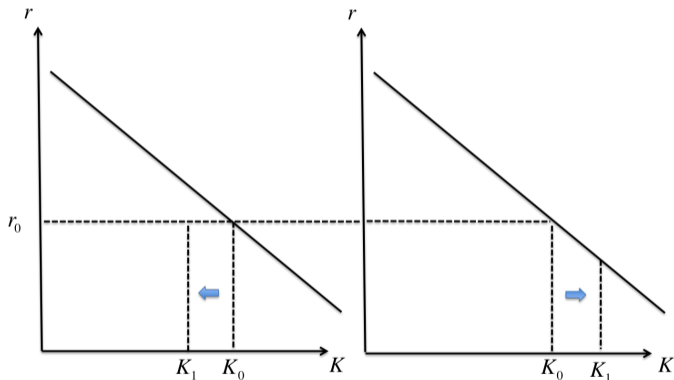
New allocation of capital

- K flows to east, lowering net returns in both
- Flows continue until after tax return is equalized across markets
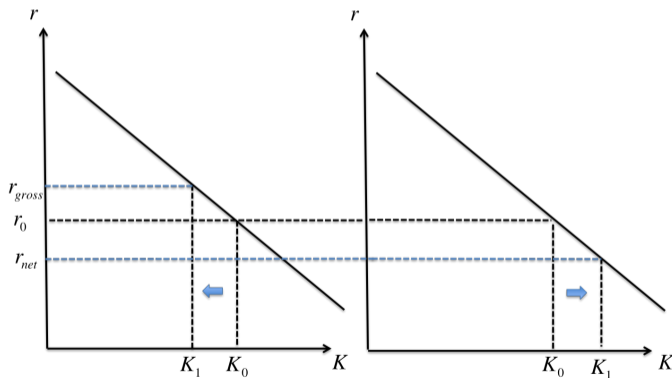
Welfare changes in each location

- Welfare in west falls by red amount
- Welfare in east increases
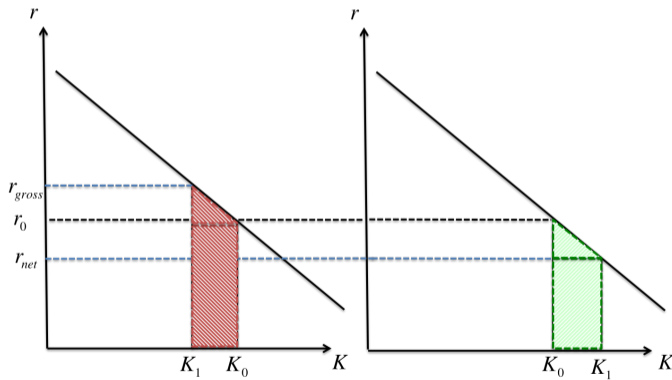
- Net welfare loss in red
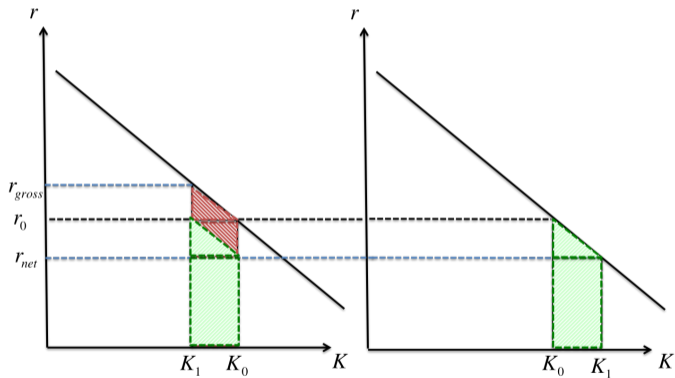
What determines size of welfare loss in this toy example?

1. Size of tax change
2. Size of market being taxed (depends on fundamentals)
3. Elasticity of demand in both regions (quantity response more generally, which depends on S and D elasticities)
4. Strength of complementarities across markets (e.g., labor market)
5. Assumptions about effects/value of government spending (assumed to be zero here)
6. Presence of existing distortions

Will formalize these ideas in the next section, but this example provides intuition for key forces in the Harberger model

Outline

Overview

1. **Goals**
   - Characterize effects of corporate tax change in a GE model
   - Who bears the burden of corporate taxes? (also capital, output taxes)

2. **Two sectors (or locations)**
   - Corporate sector produces output X
   - Non-corporate sector produces output Y

3. **Markets**
   - Capital: prices r_i, quantities K_i where $i \in \{X, Y\}$
   - Labor: prices w_i, quantities L_i
   - Goods: prices p_i, quantities X, Y

4. **Agents**
   - Workers (representative, perfectly mobile, supply 1 unit of labor)
   - Firm (representative, perfectly competitive, CRS)

5. **Equilibrium Conditions**
   - Good and factor markets clear, factor price equalization
   - Consumers max utility, firms earn zero profits

Comments

1. Harberger is workhorse analytical model: 2 sector and 2 factors
2. Fixed supply of capital and labor (short run, closed economy)
3. Key intuition is misallocation (magnitude depends on factor intensity, demand elasticities, etc)
4. Fullerton and Ta (2017) simplifies Harberger analysis (Cobb Douglas)
5. Similar to Hecksher-Ohlin model
6. When interpreting as locations not sectors, then implicitly assume no trade costs. Similarly, implicitly assumes no adjustment costs for capital and labor (so long run in that sense)
7. Abstracts from amenity or productivity effects of government spending (lump sum rebates or purchases in same share as consumers)

Fullerton and Ta (2017)
Parameterized Harberger Model with Cobb Douglas

Consumers: Preferences and Budget Constraint

Utility of representative worker is $U = X^\gamma Y^{1-\gamma}$

- X is corporate sector output
- Y is non-corporate sector output

Budget constraint is $p_x X + p_y Y = I$

- I is income, which is sum of labor and capital income
- p_i is price of output in sector i where $i \in \{X, Y\}$

Workers have fixed expenditure shares (e.g. $I\gamma$); demand for X and Y is:

$$X = \frac{I\gamma}{p_x}$$
$$Y = \frac{I(1-\gamma)}{p_y}$$

N.B. note no labor supply or saving decision

Consumers: Indirect Utility

- Indirect utility is given

$$V(p_x, p_y, I) = \left(\frac{I\gamma}{p_x}\right)^{\gamma} \left(\frac{I(1-\gamma)}{p_y}\right)^{1-\gamma} = \frac{I}{\bar{p}}$$

where $\bar{p} = \left(\frac{p_x}{\gamma}\right)^{\gamma} \left(\frac{p_y}{1-\gamma}\right)^{1-\gamma}$ is the "ideal" price index

- Inverting indirect utility (i.e., $V = \frac{I}{\bar{p}}$), gives the expenditure function $I = E$:

$$E(\bar{p}, U) = U\bar{p}$$

- So $\bar{p}$ is the price paid for each "util"

Firms maximize profits

- Corporate sector solves:

$$\max_{K_x, L_x} (1 - \tau_X) p_x X - (1 + \tau_K + \tau_{KX}) r K_x - w L_x, \text{ where } X = A K_x^\alpha L_x^{1-\alpha}$$

- where

$$\tau_X = \text{ tax on output of } X$$
$$\tau_K = \text{ tax on capital}$$
$$\tau_{KX} = \text{ tax on capital in production of } X$$

- Non-corporate sector solves:

$$\max_{K_y, L_y} p_y Y - (1 + \tau_K) r K_y - w L_y, \text{ where } Y = B K_y^\beta L_y^{1-\beta}$$

Firm optimization (and factor demand)

FOCs:

$$w = (1 - \tau_X)p_x(1 - \alpha)A\left(\frac{K_x}{L_x}\right)^{\alpha}$$

$$w = p_y(1 - \beta)B\left(\frac{K_y}{L_y}\right)^{\beta}$$

and

$$(1 + \tau_K + \tau_{KX})r = (1 - \tau_X)p_x\alpha A\left(\frac{L_x}{K_x}\right)^{1-\alpha}$$

$$(1 + \tau_K)r = p_y\beta B\left(\frac{L_y}{K_y}\right)^{1-\beta}$$

Exogenous parameters

- Taxes: τ_X, τ_K, τ_{KX}
  - τ_X is tax on corporate sector output (sales tax)
  - τ_K is tax on capital
  - τ_{KX} is tax on capital used in corporate sector

- Consumer Parameter: γ
  - γ governs importance of corporate goods for utility
  - $1 - \gamma$ governs importance of non-corporate goods for utility

- Firm Parameters: α, β, A, B
  - α is output elasticity of capital in sector X
  - $1 - \alpha$ output elasticity of labor in sector X
  - β output elasticity of capital in sector Y
  - $1 - \beta$ output elasticity of labor in sector Y
  - A and B are productivity in corp and non-corp sectors

- Endowments; K, L
  - K is total capital
  - L is total labor

Endogenous Model Outcomes and Equilibrium

Endogenous outcomes are $K_i, L_i, p_i, X, Y, w, r$:

- Capital: prices r_i, quantities K_i where $i \in \{X, Y\}$
- Labor: prices w_i, quantities L_i
- Goods: prices p_i, quantities X, Y

Given τ_X, τ_K, τ_{KX}, $\gamma, \alpha, \beta, A, B, K, L$, equilibrium is defined by prices and quantities $\{w, r, p_i, K_x, K_y, L_x, L_y, X, Y\}$ such that good and factor markets clear and firms and workers optimize.

Equilibrium: closed form expressions

In log terms, the equations are:

$$\ln X = \ln I + \ln \gamma - \ln p_x$$
$$\ln Y = \ln I + \ln(1 - \gamma) - \ln p_y$$
$$\ln K_x - \ln L_x - \ln w + \ln r = \ln \alpha - \ln(1 - \alpha) - \ln(1 + \tau_K + \tau_{KX})$$
$$\ln K_y - \ln L_y - \ln w + \ln r = \ln \beta - \ln(1 - \beta) - \ln(1 + \tau_K)$$
$$\ln X - \ln K_x + \ln p_x - \ln r = \ln(1 + \tau_K + \tau_{KX}) - \ln \alpha - \ln(1 - \tau_X)$$
$$-\ln X + \ln L_x - \ln p_x + \ln w = \ln(1 - \alpha) + \ln(1 - \tau_X)$$
$$\ln Y - \ln K_y + \ln p_y - \ln r = \ln(1 + \tau_K) - \ln \beta$$
$$-\ln Y + \ln K_y - \ln p_y + \ln w = \ln(1 - \beta)$$

where $K = K_x + K_y$ and $L = L_x + L_y$.

◄ Equations without taxes

Solutions with taxation (1/2)

- Given taxes $\tau_K, \tau_X,$ and $\tau_{KX},$ we have

$$X = A\gamma(1 - \tau_X) \left(\frac{\alpha(1 + \tau_K)}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} K \right)^\alpha$$

$$\left(\frac{(1 - \alpha)}{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)} L \right)^{1 - \alpha}$$

$$Y = B(1 - \gamma) \left(\frac{\beta(1 + \tau_K + \tau_{KX})}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} K \right)^\beta$$

$$\left(\frac{(1 - \beta)}{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)} L \right)^{1 - \beta}$$

$$K_x = \frac{\alpha\gamma(1 - \tau_X)(1 + \tau_K)}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} K$$

$$K_y = \frac{\beta(1 - \gamma)(1 + \tau_K + \tau_{KX})}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} K$$

$$L_x = \frac{(1 - \alpha)\gamma(1 - \tau_X)}{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)} L$$

Solutions with taxation (1/2)

$$L_y = \frac{(1-\beta)(1-\gamma)}{(1-\alpha)\gamma(1-\tau_X) + (1-\beta)(1-\gamma)} L$$

$$p_x = \frac{I}{A(1-\tau_X)} \left(\frac{\alpha\gamma(1-\tau_X)(1+\tau_K) + \beta(1-\gamma)(1+\tau_K+\tau_{KX})}{\alpha(1+\tau_K)K} \right)^\alpha$$

$$\left(\frac{(1-\alpha)\gamma(1-\tau_X) + (1-\beta)(1-\gamma)}{(1-\alpha)L} \right)^{1-\alpha}$$

$$p_y = \frac{I}{B} \left(\frac{\alpha\gamma(1-\tau_X)(1+\tau_K) + \beta(1-\gamma)(1+\tau_K+\tau_{KX})}{\beta(1+\tau_K+\tau_{KX})K} \right)^\beta$$

$$\left(\frac{(1-\alpha)\gamma(1-\tau_X) + (1-\beta)(1-\gamma)}{(1-\beta)L} \right)^{1-\beta}$$

$$w = \frac{I}{L} \left[(1-\alpha)\gamma(1-\tau_X) + (1-\beta)(1-\gamma) \right]$$

$$r = \frac{I}{K} \left[\frac{\alpha\gamma(1-\tau_X)(1+\tau_K) + \beta(1-\gamma)(1+\tau_K+\tau_{KX})}{(1+\tau_K)(1+\tau_K+\tau_{KX})} \right]$$

Welfare Loss from Taxation

Equivalent variation and Burden of taxation

- Equivalent variation EV is the change in wealth at initial prices that would be equivalent to the price change in terms of utility.

$$EV = E(\bar{p}^0, U^1) - \bar{I}$$
$$= \bar{p}^0 U^1 - \bar{p}^0 U^0 = \bar{p}^0 (U^1 - U^0)$$

where $\bar{p}^0$ and $\bar{p}^1$ are the "ideal" prices in period 0 and 1

- Use $-EV$ as a positive measure of tax burden, so

$$EB = -EV = \bar{p}^0 (U^0 - U^1)$$

Amount that burden exceeds tax revenues is called excess burden (Auerbach and Hines, 2002). We consider a revenue-neutral reform with a distorting tax where all revenue is returned lump sum, so net revenue is zero and thus, net loss *is* excess burden.

Average and Marginal Excess Burden

- Average Excess Burden (*AEB*) is the total welfare loss from the tax divided by the total revenue collected by the government:

$$AEB = \frac{EB}{R}$$

where $\bar{p}^0$ and $\bar{p}^1$ are the "ideal" prices in period 0 and 1

- Marginal excess burden (*MEB*) measures the effects of a small change in the tax rate on burden:

$$\Delta EB = \bar{p}^0(EB^1 - EB^2)$$
$$MEB = \frac{\Delta EB}{\Delta R}$$

N.B. See Hendren's recent TPE paper for more detailed discussion

Understanding Equilibrium:
Graphical and quantitative analysis

Understanding Equilibrium: Graphs and numerical example

- There are a lot of moving parts
- Helpful to think about relative factor markets (relative prices and relative quantities) in the two sectors
- Will start with demand side, then supply side, then analyze equilibrium graphically pre and post taxes
- Will work with a calibrated version of the model to do quantitative analysis

Taking ratios of each sector's FOCs gives:

$$\frac{w}{r} = \frac{(1-\alpha)}{\alpha} \left(\frac{L_x}{K_x}\right)^{-1} (1 + \tau_K + \tau_{KX}) \tag{21}$$

$$\frac{w}{r} = \frac{(1-\beta)}{\beta} \left(\frac{L_y}{K_y}\right)^{-1} (1 + \tau_K) \tag{22}$$

Relative factor supply

Recall

$$L = L_x + L_y$$
$$K = K_x + K_y$$

Thus, the economy-wide labor capital ratio is:

$$\frac{L}{K} = \frac{L_x}{K} + \frac{L_y}{K}$$
$$\frac{L}{K} = \frac{L_x}{K_x}\left(\frac{K_x}{K}\right) + \frac{L_y}{K_y}\left(\frac{K_y}{K}\right) \tag{23}$$

This says that overall labor to capital ratio is a weighted average of the labor to capital ratio in both sectors

Factor Market Prices and Quantities

We can invert 21 and 22 to get L_x/K_x and L_y/K_y as functions of w/r. Then

$$\frac{L}{K} = \left(\frac{w}{r}\right)^{(-1)} \left(\frac{(1+\tau_k+\tau_{kx})(1-\alpha)}{\alpha} \frac{K_x}{K} + \frac{(1+\tau_k)(1-\beta)}{\beta} \frac{K_y}{K} \right) \tag{24}$$

In equilibrium, we found

$$\frac{K_x}{K} = \frac{\alpha\gamma(1-\tau_X)(1+\tau_K)}{\alpha\gamma(1-\tau_X)(1+\tau_K) + \beta(1-\gamma)(1+\tau_K+\tau_{KX})} \tag{25}$$

$$\frac{K_y}{K} = \frac{\beta(1-\gamma)(1+\tau_K+\tau_{KX})}{\alpha\gamma(1-\tau_X)(1+\tau_K) + \beta(1-\gamma)(1+\tau_K+\tau_{KX})} \tag{26}$$

Numerical Example

- X and Y produced given functions

$$X = AK_x^{.6}L_x^{.4}$$
$$Y = BK_y^{.2}L_y^{.8}$$

- Identical households have utility:

$$U = X^{.5}Y^{.5}$$

- Fixed[4] level of income $I = 2,400$
- Demand for X and Y is given by:

$$X = \frac{2400(.5)}{p_x} = \frac{1200}{p_x}$$
$$Y = \frac{2400(.5)}{p_y} = \frac{1200}{p_y}$$

- Assume unity of prices in the initial state ($p_x = p_y = r = w = 1$)

Initial quantities and prices ($\tau_K = \tau_{KX} = \tau_X = 0$)

- With this parameterization of utility and technology, we have:

	Value
L_x	480
L_y	960
K_x	720
K_y	240
X	1200
Y	1200
p_x	1
p_y	1
r	1
w	1

- Use the values above to derive $A \approx 1.96$ and $B \approx 1.69$ (given output, inputs, and production function)
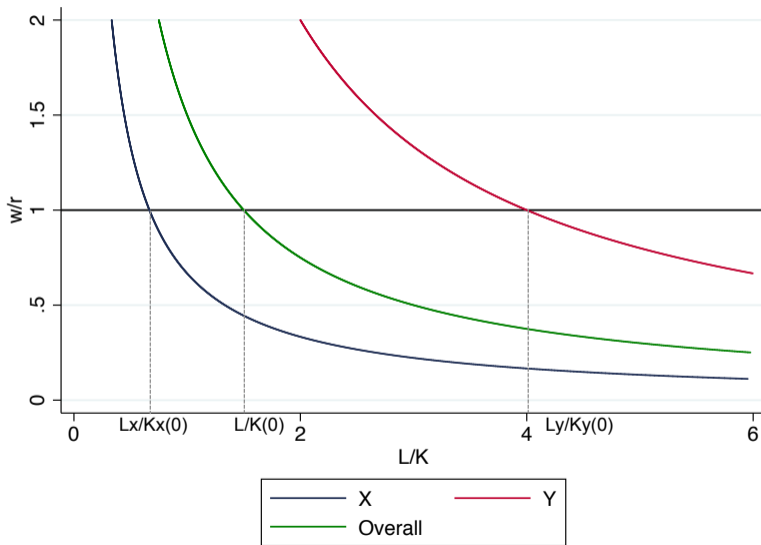
1. τ_X reduces demand for X

2. We will have factors move from producing X to producing Y until prices and quantities re-equilibrate

3. Specifically, since $w_x = w_y \Rightarrow (1 + \tau_x)p_x MPL_x = p_y MPL_y$, we need a combination of lower p_x and higher MPL_x (and thus lower factor demand in x) and/or higher p_y and lower MPL_y

4. The movement of both factors to Y increases the weight of the non-corporate sector in labor and capital demand (see eq 24 and dashed green line in next slide)

5. Since the non-corporate sector is relatively labor intensive, total relative labor demand increases

6. Hence, the equilibrium wage to rental ratio increases

Tax on Corporate Output: Prices and Quantities

Panel A: Allocations and Prices			
	$t_X = 0$	$t_X = .3$	$t_X = .31$
L_x	480	373.333	369.368
L_y	960	1,066.667	1,070.632
K_x	720	650.323	647.296
K_y	240	309.677	312.704
X	1,200	1,020.942	1,013.750
Y	1,200	1,373.811	1,380.577
p_x	1	1.175	1.184
p_y	1	0.874	0.869
r	1	0.775	0.768
w	1	0.900	0.897
w/r	1	1.161	1.168
L_x/K_x	0.667	0.574	0.571
L_y/K_y	4	3.444	3.424

Panel B: Exact Measures of Welfare			
	$t_X = 0$	$t_X = .3$	$t_X = .31$
$\bar{p}^0$	2		
$\bar{p}$	2	2.026	2.029
U	1,200	1,184.306	1,183.030
EB	0	31.387	33.940
R	0	360	372
AEB		0.087	0.091
MEB			0.213

Effect of Tax on Capital

1. Suppose a tax on all capital: $\tau_K = .3$, and $\tau_{KX} = \tau_X = 0$

2. Both sectors face tax on capital, so capital allocation across sectors does not change (see 25 and 26 in which the $(1 + \tau_k)$ terms cancel)

3. $(1 + \tau_k)$ increases relative labor demand symmetrically in eq 24 in both sectors (i.e., it shifs up L_i/K_i), so factor allocation stays constant and all adjustment is through relative prices

4. In this case, capital fully bears the burden of the tax (i.e., w/r rises by 30% to offset tax increase)

N.B. remember that in these examples, the *overall* stock of capital is fixed. In practice, investment and firm creation respond to taxes. A key question is how much they respond
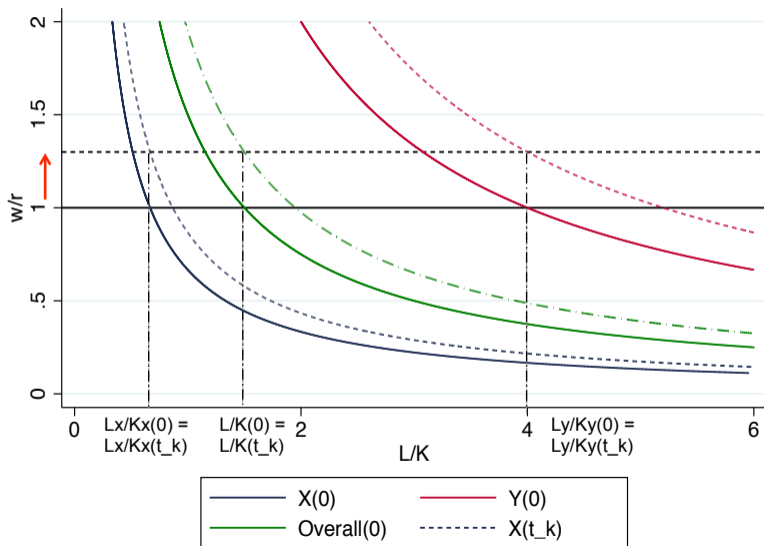
Tax on Capital: Allocation and Quantities

Panel A: Allocations and Prices			
	$t_K=0$	$t_K=.3$	$t_K=.31$
L_x	480	480	480
L_y	960	960	960
K_x	720	720	720
K_y	240	240	240
X	1,200	1,200	1,200
Y	1,200	1,200	1,200
p_x	1	1	1
p_y	1	1	1
r	1	0.769	0.763
w	1	1	1
w/r	1	1.300	1.310

Panel B: Exact Measures of Welfare			
	$t_K = 0$	$t_K = .3$	$t_K = .31$
$\bar{p}^0$	2		
$\bar{p}$	2	2	2
U	1,200	1,200	1,200
EB	0	0	0
R	0	221.538	227.176
AEB		0	0
MEB			0

Effect of Tax on Corporate Capital

1. Now suppose a tax on corporate capital, $\tau_{KX} = .3$

2. Corporate sector demands less capital ($r_x = \frac{p_x MPK_x}{1+\tau_{KX}}$), so capital flows from corporate to non-corporate sector (see eq 25 and 26)

3. Lower capital allocation to producing X increases the weight of the non-corporate sector in labor and capital demand (see eq 24 and dashed green line in next slide)

4. Causes misallocation (too much K_y and thus, too much Y, not enough X), which reduces welfare as in prior example
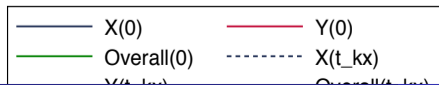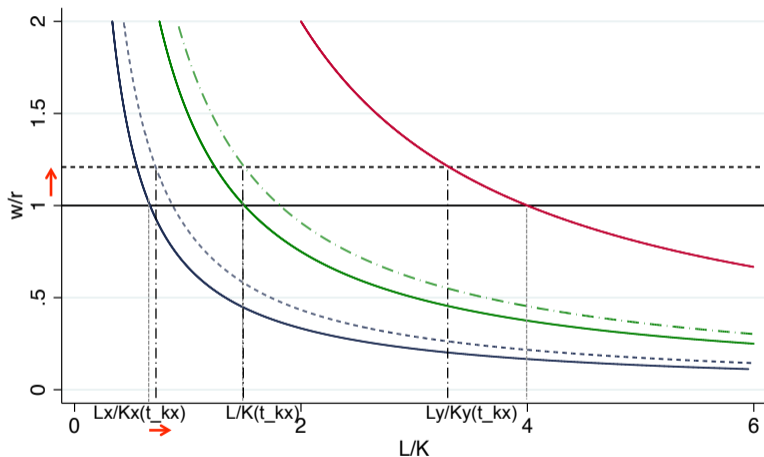
Tax on Corporate Capital: Allocation and Quantities

Panel A: Allocations and Prices			
	$t_{KX} = 0$	$t_{KX} = .3$	$t_{KX} = .31$
L_x	480	480	480
L_y	960	960	960
K_x	720	669.767	668.214
K_y	240	290.233	291.786
X	1,200	1,149.043	1,147.442
Y	1,200	1,246.488	1,247.820
p_x	1	1.044	1.046
p_y	1	0.963	0.962
r	1	0.827	0.823
w	1	1	0.897
w/r	1	1.209	1.090
L_x/K_x	0.667	0.717	0.718
L_y/K_y	4	3.308	3.290
L/K	1.500	1.500	1.500

Panel B: Exact Measures of Welfare			
	$t_{KX} = 0$	$t_{KX} = .3$	$t_{KX} = .31$
$\bar{p}^0$	2		
$\bar{p}$	2	2.005	2.006
U	1,200	1,174.245	1,173.427
EB	0	51.510	53.146
R	0	166.154	170.382
AEB		0.310	0.312
MEB			0.387

Harberger Model

Ten equations needed for equilibrium are:

$$p_x X + p_y Y = wL + rK \tag{27}$$

$$MRS_{XY} = \frac{p_x(1 + \tau_X)}{p_y} \tag{28}$$

$$c_x(w, r(1 + \tau_K + \tau_{KX})) = p_x \tag{29}$$

$$c_y(w, r(1 + \tau_K)) = p_y \tag{30}$$

$$w = p_x F_{xL} \tag{31}$$

$$w = p_y F_{yL} \tag{32}$$

$$r(1 + \tau_K + \tau_{KX}) = p_x(1 - \tau_X)F_{xK} \tag{33}$$

$$r = p_y F_{yK} \tag{34}$$

$$K = K_x + K_y \tag{35}$$

$$L = L_x + L_y \tag{36}$$

Definitions

- Share of income spent on X and Y:

$$s_x \equiv \frac{p_x X}{p_x X + p_y Y}, \ \ s_y \equiv \frac{p_y Y}{p_x X + p_y Y}, \ \ s_x + s_y = 1$$

- Share of income from labor and capital:

$$s_w \equiv \frac{wL}{wL + rK}, \ \ s_r \equiv \frac{rK}{wL + rK}$$

- Cost shares in production of X and Y:

$$\theta_L \equiv \frac{wL_x}{wL_x + rK_x}, \ \theta_K \equiv \frac{rK_x}{wL_x + rK_x}, \ \ \theta_x + \theta_y = 1$$

$$\phi_L \equiv \frac{wL_y}{wL_y + rK_y}, \ \phi_K \equiv \frac{rK_y}{wL_y + rK_y}, \ \ \phi_x + \phi_y = 1$$

Definitions

- Share of labor and capital used to produce X:

$$\lambda_L \equiv \frac{L_x}{L}, \;\; \lambda_K \equiv \frac{K_x}{K}$$

- By Euler's Theorem and CRS, we also have:

$$p_x X = w L_x + r K_x, \;\; p_y Y = w L_y + r K_y$$

$$\Rightarrow \lambda_L = \frac{s_x \theta_L}{s_x \theta_L + s_y \phi_L} = \frac{s_x (1 - \theta_K)}{1 - s_x \theta_K - s_y \phi_K}$$

$$\Rightarrow \lambda_K = \frac{s_x \theta_K}{s_x \theta_K + s_y \phi_K} = \frac{s_x (1 - \theta_L)}{1 - s_x \theta_L - s_y \phi_L}$$

Log-Linearization

$$s_x(\hat{p_x} + \hat{X}) + s_y(\hat{p_y} + \hat{Y}) = s_w\hat{w} + s_r\hat{R}$$

$$\hat{X} - \hat{Y} = \sigma_D(\hat{p_y} - \hat{p_x} - d\tau_X)$$

$$\hat{p_x} = \theta_L\hat{w} + \theta_K(\hat{r} + d\tau_K + d\tau_{KX})$$

$$\hat{p_y} = \phi_L\hat{w} + \phi_K(\hat{r} + d\tau_K)$$

$$\lambda_L\hat{L_x} + (1 - \lambda_L)\hat{L_y} = 0$$

$$\lambda_K\hat{K_x} + (1 - \lambda_K)\hat{K_y} = 0$$

$$\hat{L_x} = \hat{X} + \theta_K\sigma_X(\hat{r} + d\tau_K + d\tau_{KX} - \hat{w})$$

$$\hat{K_x} = \hat{X} + \theta_L\sigma_X(\hat{w} - d\tau_K - d\tau_{KX} - \hat{r})$$

$$\hat{L_y} = \hat{Y} + \phi_K\sigma_Y(\hat{r} + d\tau_K - \hat{w})$$

$$\hat{K_y} = \hat{Y} + \phi_L\sigma_Y(\hat{w} - d\tau_K - \hat{r})$$

Matrix Form of the System of Linear Equations

$$\begin{bmatrix} \sigma_D & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -\theta_L & -\theta_K & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi_L & -\phi_K & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_L & 0 & 1-\lambda_L & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_K & 0 & 1-\lambda_K \\ 0 & \theta_K\sigma_X & -\theta_K\sigma_X & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -\theta_L\sigma_X & \theta_L\sigma_X & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \phi_K\sigma_Y & -\phi_K\sigma_Y & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & -\phi_L\sigma_Y & \phi_L\sigma_Y & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{p}_x \\ \hat{w} \\ \hat{r} \\ \hat{X} \\ \hat{Y} \\ \hat{L}_x \\ \hat{K}_x \\ \hat{L}_y \\ \hat{K}_y \end{bmatrix} = \begin{bmatrix} -\sigma_X \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} d\tau_X + \begin{bmatrix} 0 \\ -\theta_K \\ 0 \\ 0 \\ 0 \\ \theta_K\sigma_X \\ -\theta_L\sigma_X \\ 0 \\ 0 \end{bmatrix} d\tau_{KX}$$

where we eliminated equation 1 by Walras law and normalized $p_y = 1$, so $\hat{p}_y = 0$

Two Main Effects of Taxing K_x

1. **Substitution effects:** capital bears incidence

2. **Output effects:** capital may not bear all incidence

Substitution effects

- Tax on K_x shifts production in X away from K so aggregate demand for K goes down

- Because total K is fixed, r falls $\rightarrow K$ bears some of the burden

Another intuition for this is that capital is misallocated across sectors, which lowers r and rK

Output effects

- Tax on K_x makes X more expensive

- Demand shifts to Y

- *Case 1: $K_x/L_x > K_y/L_y$ (X: cars, Y: bikes)*
  - X more capital intensive $\rightarrow$ lower aggregate demand for K
  - Output + subst. effect: K bears the burden of the tax

- *Case 2: $K_x/L_x < K_y/L_y$ (X: bikes, Y: cars)*
  - X less capital intensive $\rightarrow$ higher aggregate demand for K
  - Subst. and output effects have opposite signs $\rightarrow$ labor may bear some the tax

Harberger showed that under a variety of reasonable assumptions, capital bears exactly 100 percent of the tax. Note that this is the burden on all capital – as capital flees the corporate sector, it depresses returns in the noncorporate sector as well. Both the realism of the model and the characterization of the corporate income tax as an extra tax on capital in the corporate sector are subject to question, as discussed in considerable detail by the subsequent literature on the effects of the corporate tax. – Alan Auerbach

See Auerbach TPE paper on who bears the corporate tax for more details

Appendix:

In log terms, the equations are:

$$\ln X = \ln A + \alpha \ln K_x + (1 - \alpha) \ln L_x$$
$$= \ln I + \ln \gamma - \ln p_x$$
$$\ln Y = \ln B + \beta \ln K_y + (1 - \beta) \ln L_y$$
$$= \ln I + \ln(1 - \gamma) - \ln p_y$$
$$\ln w = \ln p_x + \ln(1 - \alpha) + \ln A + \alpha(\ln K_x - \ln L_x)$$
$$= \ln p_y + \ln(1 - \beta) + \ln B + \beta(\ln K_y - \ln L_y)$$
$$\ln r = \ln p_x + \ln \alpha + \ln A + (1 - \alpha)(\ln L_x - \ln K_x)$$
$$= \ln p_y + \ln \beta + \ln B + (1 - \beta)(\ln L_y - \ln K_y)$$

where $K = K_x + K_y$ and $L = L_x + L_y$.

Solutions in initial equilibrium without taxes (1/2)

- Solving for the system of equations, quantities are:

$$X = A\gamma \left(\frac{\alpha K}{\alpha\gamma + \beta(1-\gamma)} \right)^{\alpha} \left(\frac{(1-\alpha)L}{(1-\alpha)\gamma + (1-\beta)(1-\gamma)} \right)^{1-\alpha}$$

$$Y = B(1-\gamma) \left(\frac{\beta K}{\alpha\gamma + \beta(1-\gamma)} \right)^{\beta} \left(\frac{(1-\beta)L}{(1-\alpha)\gamma + (1-\beta)(1-\gamma)} \right)^{1-\beta}$$

$$K_x = \frac{\alpha\gamma}{\alpha\gamma + \beta(1-\gamma)} K$$

$$K_y = \frac{\beta(1-\gamma)}{\alpha\gamma + \beta(1-\gamma)} K$$

$$L_x = \frac{(1-\alpha)\gamma}{(1-\alpha)\gamma + (1-\beta)(1-\gamma)} L$$

$$L_y = \frac{(1-\beta)(1-\gamma)}{(1-\alpha)\gamma + (1-\beta)(1-\gamma)} L$$

Prices are:

$$p_x = \frac{I}{A} \left(\frac{\alpha\gamma + \beta(1-\gamma)}{\alpha K} \right)^\alpha \left(\frac{(1-\alpha)\gamma + (1-\beta)(1-\gamma)}{(1-\alpha)L} \right)^{1-\alpha}$$

$$p_y = \frac{I}{B} \left(\frac{\alpha\gamma + \beta(1-\gamma)}{\beta K} \right)^\beta \left(\frac{(1-\alpha)\gamma + (1-\beta)(1-\gamma)}{(1-\beta)L} \right)^{1-\beta}$$

$$w = \frac{I}{L} \left((1-\alpha)\gamma + (1-\beta)(1-\gamma) \right)$$
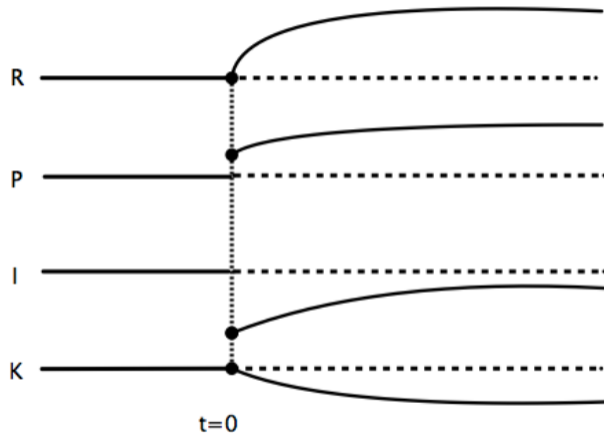
$$r = \frac{I}{K} \left(\alpha\gamma + \beta(1-\gamma) \right)$$

‹ Back

Suppose the economy is in steady state and suddenly the costs of new construction increase.

Suppose the economy is in steady state and suddenly the costs of new construction increase.

- The main impact is on the asset market. Supply curve shifts left.
- Lower supply decreases new construction
- The lower stock of capital causes rents to rise
- Rising rents and the prospects of future higher rents (remember long-run rents are higher) cause the price to rise
- As prices rise over time, construction rebounds

Decline in the interest rate

Suppose the economy is in steady state and suddenly the interest rate r declines.

Decline in the interest rate

Suppose the economy is in steady state and suddenly the interest rate r declines.

- Lower $r \Rightarrow$ future streams of payments are more valuable, so P will jump up
- The jump in asset prices causes investment to jump up
- More investment increases the capital stock
- Higher capital stocks start to decrease rental rates
- Higher rental rates decrease asset prices
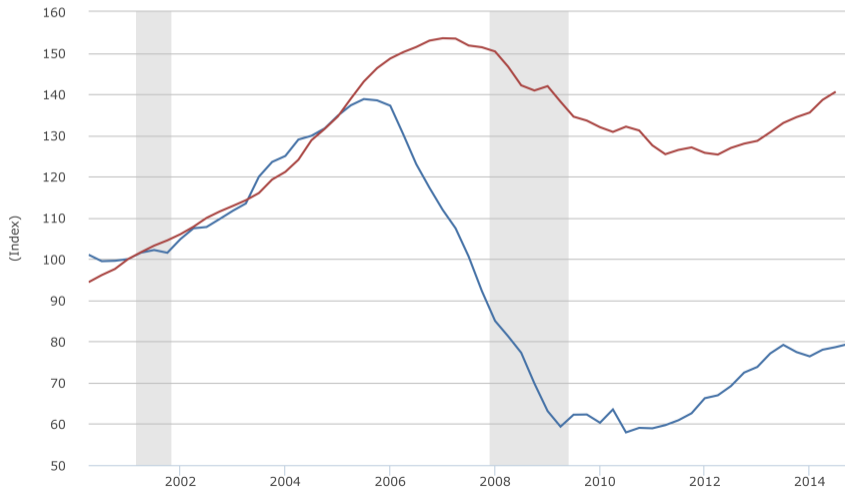- We will end at a steady state with higher K^* and lower R^*

Back

Decline in the interest rate
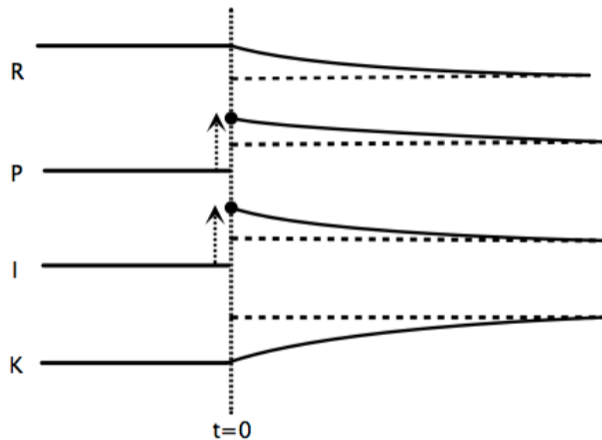
US House Prices and Residential Investment

FRED📈

— Real Private Residential Fixed Investment, 2001:Q1=100
— All-Transactions House Price Index for the United States, 2001:Q1=100

Was this an irrational bubble?

US House Prices and Residential Investment

What did we see in the **housing boom** in the 2000s?

What did we see in the **housing boom** in the 2000s?

- Low interest rates and high capital prices
- Therefore, housing services are cheap to use and investment will increase as $S(P)$ increases with P
- Suppose people expected higher future demand
- More downward pressure on rental prices, higher capital prices, more construction

What did we see in the **housing bust** in the late 2000s?

US House Prices and Residential Investment

What did we see in the **housing bust** in the late 2000s?

- We've build up a large housing stock
- Suppose now the anticipated increase in demand never comes
- ⇒ Falling house prices
- ⇒ Big decline in investment
- ⇒ Also makes consuming housing services more expensive
- ⇒ Things will adjust as housing stock goes back to the steady state

Was this an irrational bubble?

Observing a crash in the housing market does not tell us whether (1) there were rational expectations about future demand (coupled with low interest rates) or (2) a bubble (that could not be justified by expectations). ◄ Back